

***Q* factor estimation based on frequency-weighted-exponential function of power spectrum**

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Summary

The quality factor Q is a powerful tool for hydrocarbon detection and reservoir characterization in seismic data processing and interpretation. There are various methods for Q estimation, such as centroid frequency shift method (CFS), frequency weighted exponential method (FWE) and peak frequency shift (PFS). However each method has its own limitations. The frequency-weighted-exponential (FWE) function can fit various asymmetric amplitude spectrum by two key parameters: symmetry index and bandwidth factor. In this paper, we use the mean frequency and its deviation of power spectrum to calculate the two parameters of FWE function. In this way, the new FWE function can fit both asymmetrical and symmetrical amplitude spectrum accurately. Meanwhile we recalculate the symmetry index by the average of the symmetry indexes of source wavelet and received wavelet to estimate Q factor. Synthetic data test proves that this proposed method is more robust to random noise compared with the existing methods.

Introduction

Due to the viscoelastic attenuation and heterogeneity of medium, seismic waves suffer energy dissipation and waveform distortion when they propagate through the subsurface. Understanding, estimating and compensating for the attenuation plays important roles in seismic data processing, such as improving the resolution of seismic images, better interpreting the effects of AVO, and inverting for material properties. Q factor is usually used to quantify the attenuation and it is a useful tool for hydrocarbon detection and reservoir characterization. Therefore a reliable Q factor estimating method is needed in the seismic exploration.

Q factor estimation measurement of subsurface medium has been an important research topic in exploration geophysics. Among the existing methods for Q factor estimation, frequency-domain methods are commonly used, including the logarithm spectral ratio method (LSR) (Hauge, 1981), the centroid frequency shift method (CFS) (Quan and Harris, 1997), and the peak frequency shift method (PFS) (Zhang and Ulrych, 2002). However LSR method is easily affected by effective bandwidth chosen to fit a slope. Subsequently, Q estimation methods based on the frequency shift have been proposed. CFS method and PFS method are proposed under the assumption of particular source wavelet types so that the application of these methods are restricted. Hu et al. (2011) designed the

frequency-weighted-exponential function (FWE), which can be used to fit various asymmetrical amplitude spectra by two separate parameters: the symmetry index and the bandwidth factor, and they introduce it into attenuation tomography. Based on this, Li and Liu (2015) proposed a new method for Q factor estimation by using the two parameters. To meet the growing challenge of seismic exploration, a more accurate and applicable method to estimate the quality factor Q need to be developed.

When we use FWE function to fit various asymmetrical amplitude spectrum, symmetry index and bandwidth factor are the two key parameters. In this paper, we use the mean frequency and its deviation of power spectrum to derive the two key parameters. Meanwhile, we propose an improved method to recalculate the symmetry index considering that the amplitude spectrum of receiver wavelet will be changed after attenuation. Then we can build the relationship between the FWE function of power spectrum with the Q factor. Compared with the conventional FWE function, it has better tolerance to random noise and can be applied to fit both symmetrical amplitude spectrum and asymmetrical amplitude spectrum. Synthetic data test is carried out to prove that this method is more accurate and applicable over conventional approaches.

Method

First we introduce the conventional FWE method (Hu et al., 2011) briefly. The spectrum of the source wavelet can be express as

$$S(f) = Af^n \exp\left(-\frac{f}{f_0}\right) \quad (1)$$

Where A is a constant for amplitude scaling; n is the symmetry index controlling the symmetrical property; f_0 is the bandwidth factor controlling the bandwidth. In the conventional FWE method, the centroid frequency of $S(f)$ can be derived as

$$f_s = \frac{\int_0^\infty fS(f)df}{\int_0^\infty S(f)df} = (n+1)f_0 \quad (2)$$

According to the Aki-Richards theory, the amplitude spectrum of received signal can be written as

$$R(f) = S(f) \exp\left(-\frac{\pi f \Delta t}{Q}\right) \quad (3)$$

FWE function of power spectrum

Where the exponential function describes the seismic attenuation effect. Δt is the travel time. Then bring equation (1) to the equation (3), we can get

$$R(f) = Af^n \exp\left(-f\left(\frac{1}{f_0} + \frac{\pi\Delta t}{Q}\right)\right) \quad (4)$$

The centroid frequency of the received wavelet is

$$f_R = \frac{\int_0^\infty fR(f)df}{\int_0^\infty R(f)df} = \frac{n+1}{\frac{\pi\Delta t}{Q} + \frac{1}{f_0}} \quad (5)$$

Then the centroid frequency shift between the source wavelet amplitude spectrum and the received signal amplitude spectrum can be easily obtained as

$$\Delta f = f_s - f_R = (n+1) \left(f_0 - \frac{1}{\frac{\pi\Delta t}{Q} + \frac{1}{f_0}} \right) \quad (6)$$

Hence we can estimate the Q factor by this centroid frequency shift of amplitude spectrum as follow

$$Q = \frac{\pi\Delta t f_s f_R}{(n+1)(f_s - f_R)} \quad (7)$$

Formular (1) to (7) describe the conventional method FWE method. It is simple and easy to apply to real data. However, this method will introduce large errors when the data is noisy.

so we propose to an improved method to estimate Q factor based on FWE method using the mean frequency and its deviation of power spectrum instead of amplitude spectrum.

First the mean frequency and its deviation can be evaluated from the power spectrum $S(f)^2$ using integrals as

$$f_s = \frac{\int_0^\infty fS(f)^2 df}{\int_0^\infty S(f)^2 df} = (2n+1) \frac{f_0}{2} \quad (8)$$

$$\sigma_s^2 = \frac{\int_0^\infty (f - f_s)^2 S(f)^2 df}{\int_0^\infty S(f)^2 df} = (2n+1) \left(\frac{f_0}{2}\right)^2 \quad (9)$$

Then we can obtain a new simple form of f_0 and n from the statistical properties (f_s and σ_s^2) as follow

$$n = \frac{1}{2} \left(\frac{f_s^2}{\sigma_s^2} - 1 \right) \quad (10)$$

$$f_0 = \frac{2\sigma_s^2}{f_s} \quad (11)$$

Once the mean frequency f_s and the deviation σ_s^2 are evaluated from the power spectrum of the source wavelet, we can obtain the two key parameters (n and f_0) of the FWE function.

Then we use the mentioned method and conventional FWE method to fit three different wavelet types. In the Figure 1(a), there is a Ricker wavelet with $f_0=50$ Hz, $\sigma_s^2=200$ and its amplitude spectrum is the blue curve in the Figure 1(b). The red curve in the Figure 1(b) is the amplitude spectrum fitted by the equation (10) and equation (11), while the yellow curve is fitted by the conventional FWE function. The correlation coefficient between these two curves with the blue one is denoted as c in the Figure 1(b). In the similar way, we can fit the Ricker-like wavelet (in the Figure 1(c)) and the Gaussian function (in the Figure 1(e)) using these analytical expressions mentioned. The results are carried out (in the Figure 1(d) and 1(f)) to testify that the FWE function of power spectrum can fit various source wavelet types perfectly.

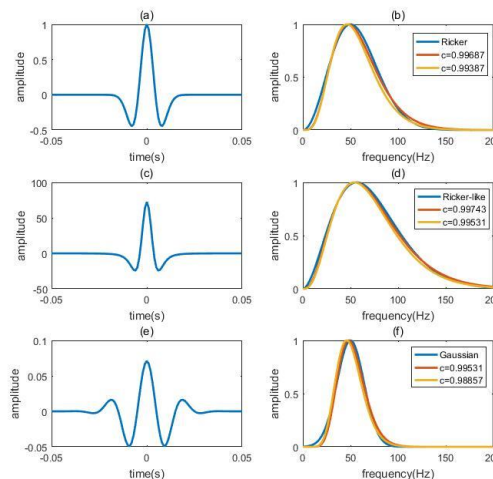


Figure 1 amplitude-spectrum are fitted by the FWE function of power spectrum and conventional FWE function and c is the correlation coefficient. (a) is the Ricker wavelet and its amplitude spectrum (b); (c) is the Ricker-like wavelet and its amplitude spectrum (d); (e) is the Gaussian function and its amplitude spectrum (f).

Next calculate the mean frequency f_R and its deviation σ_R^2 of the received wavelet $R(f)$ as follow

FWE function of power spectrum

$$f_R = \frac{\int_0^\infty fR(f)^2 df}{\int_0^\infty R(f)^2 df} = (2n+1) \frac{1}{2\left(\frac{1}{f_0} + \frac{\pi\Delta t}{Q}\right)} \quad (12)$$

$$\sigma_R^2 = \frac{\int_0^\infty (f - f_R)R(f)^2 df}{\int_0^\infty R(f)^2 df} = (2n+1) \left(\frac{1}{2\left(\frac{1}{f_0} + \frac{\pi\Delta t}{Q}\right)}\right)^2 \quad (13)$$

From the mean frequency shift between the source amplitude $S(f)$ and the received amplitude $R(f)$ after attenuation, we can obtain

$$\Delta f = f_S - f_R = \frac{2n+1}{2} \left(f_0 - \frac{1}{\frac{1}{f_0} + \frac{\pi\Delta t}{Q}} \right) \quad (14)$$

So we get a new equation for estimating Q factor as follow

$$Q = \frac{2\pi f_S f_R \Delta t}{(2n+1)(f_S - f_R)} \quad (15)$$

However, according to the equation (4), the symmetry index n remaining constant after attenuation is the prerequisite for estimating Q factor using the equation (15). Especially, in the field seismic data, this condition is difficult to be satisfied. To solve this problem, we propose to recalculate the parameter n shown as the follow equation.

$$\bar{n} = (n_S + n_R) / 2 \quad (16)$$

Where n_S and n_R are the symmetry indexed of the source wavelet and the received wavelet respectively. Therefore, insert the new n into equation (15) and get a modified equation

$$\bar{Q} = \frac{2\pi f_S f_R \Delta t}{(n_S + n_R + 1)(f_S - f_R)} \quad (17)$$

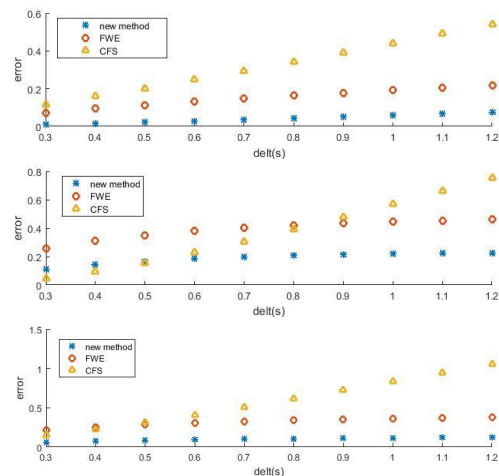
This improved method for estimating Q factor using the equation (17) is based on mean frequency shift and frequency-weighted-exponential (FWE) function of power spectrum. To avoid the limitation of the existing methods, we introduce the mean frequency and its deviation to redefine the parameters of FWE function and recalculate the symmetry index n by averaging the symmetry indexes of the source wavelet and the received wavelet. Then we will use synthetic data to test this proposed approach in the next section.

Example

In order to examine the effectiveness and stability of this proposed method, we use different types of source wavelet to estimate the Q factor comparing with CFS method and FWE method. First of all, we make the Ricker wavelet as

the source wavelet with $f_m=50$ propagate in a medium where the constant Q factor is 50 and the travel time increase from 300ms to 1200ms. The results show the relative error of estimating Q factor using these three different methods in the Figure 2(a). Next we choose a source wavelet in a shape of FWE function with $n=2, f_0=50$ propagating in the same medium and the relative error of estimating Q factor are shown in the Figure 2(b). In the Figure 2(c) and 2(d), we choose Ricker-like wavelet with $m=2, k=1.2$ and $m=1.5, k=1.2$ respectively to estimate Q factor using these three different methods. From these results we can see that the relative error of the CFS method increase rapidly as the travel time increases, because its accuracy and stability depend on the amplitude spectrum. That is to say, CFS method is hardly applicable in field data. In the Figure3, we choose Ricker-like wavelet with different parameters when propagating in the medium where the Q factor varies from 30 to 100 and the travel time increase from 300ms to 2200ms. The best outputs are given by the new method which proves that it is accurate and applicable whether the source wavelet is standard FWE function or not.

Then we add Gaussian random noise to the model in the Figure 2(a), the results of estimating Q factor using new method compared with FWE function are shown in the Figure 4(a) with SNR =50db and Figure 4(b) with SNR=20db respectively. Compared with FWE method, the new method we proposed performs more robustly especially when the attenuation and noise become stronger. The results of estimating Q factor using our improved method are considerable closest to the true value and present high robustness in the different condition. Therefore we can see that this proposed method is both reliable and robust.



FWE function of power spectrum

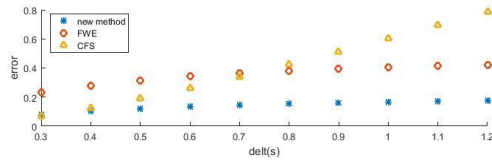


Figure 2 shows the relative error of estimating Q factor using CFS method, FWE method and the new method when the source wavelets are different types. There are Ricker wavelet (a), FWE function (b) and the Ricker-like wavelet (c), (d).

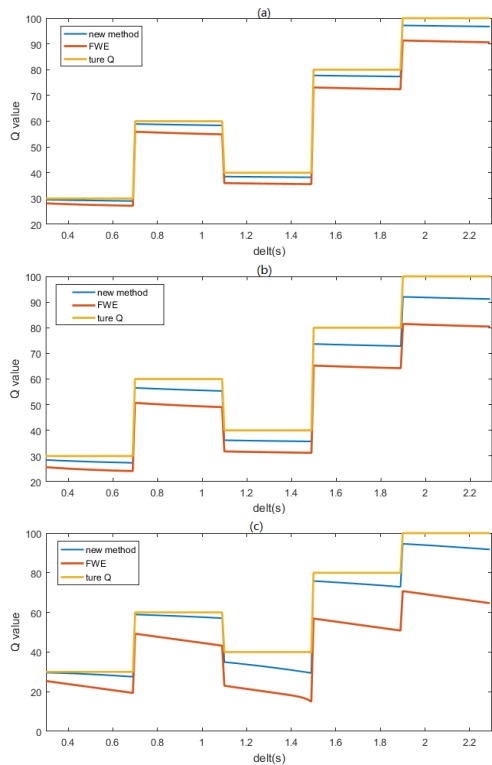


Figure 3 shows the results of estimating Q factor using the new method and FWE method when the source wavelet is Ricker-like wavelet with different parameters. (a) is the Ricker-like wavelet with parameter $m=1.8, k=1.2$; (b) is the Ricker-like wavelet with parameter $m=0.9, k=1.4$; (c) is the Ricker-like wavelet with parameter $m=1.5, k=1.8$;

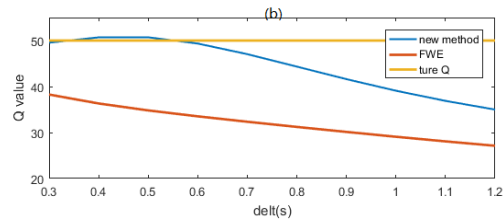
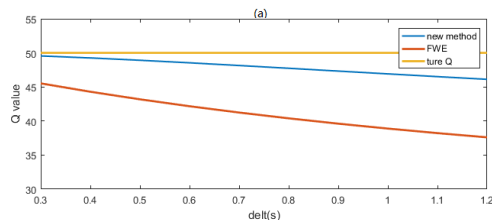


Figure 4 shows the results of estimating Q factor using new method compared with FWE function when the source wavelet is Ricker wavelet with $SNR=50db$ and $SNR=20db$ respectively.

Conclusion

In this paper, we proposed a novel method for Q estimation based on the FWE function. In practice, once the statistical properties (the mean frequency and deviation) are evaluated from the seismic data of power spectrum, the symmetry index and the characteristic frequency of FWE function can be uniquely determined. To fit more source wavelet whose amplitude spectrum's shape is nonstandard FWE function, we recalculate symmetry property by using the average symmetry indexes of source wavelet and received wavelet. Subsequently we can estimate Q factor based on the mean frequency shift and the new symmetry property. On the one hand this proposed method overcomes the disadvantage of CFS method which depends on the assumption of particular source wavelet types. On the other hand, this new method improves the accuracy and robustness and can be applied to the symmetrical source amplitude spectrum compared with FWE method especially when the attenuation is strong. Synthetic data demonstrate that the new method is applicable in hydrocarbon detection and reservoir characterization.

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REFERENCES

- Hu, W., J. Liu, L. Bear, and C. Marcinkovich, 2011, A robust and accurate seismic attenuation tomography algorithm: 81th Annual International Meeting, SEG, Expanded Abstract, 2727–2731, <https://doi.org/10.1190/1.3627760>.
- Quan, Y., and J. M. Harris, 1997, Seismic attenuation tomography using the frequency shift method: *Geophysics*, **62**, 895–905, <https://doi.org/10.1190/1.1444197>.
- Chuanhui, L., and X. Liu, 2015, A new method for interval Q-factor inversion from seismic reflection data: *Geophysics*, **80**, no. 6, R361–R373, <https://doi.org/10.1190/geo2014-0446.1>.
- Hauge, P. S., 1981, Measurements of attenuation from vertical seismic profiles: *Geophysics*, **46**, 1548–1558, <https://doi.org/10.1190/1.1441161>.
- Zhang, C., and T. J. Ulrich, 2002, Estimation of Q-factors from CMP records: *Geophysics*, **67**, 1542–1547, <https://doi.org/10.1190/1.1512799>.